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The purpose of this contribution is to stress the great importance of polarization observables for the study of nucleon resonances, in particular in the case of deuteron and proton probes. Polarization observables play a unique role in signing the reaction mechanism and bring new information in the microscopic description of baryon resonances. We will calculate the relevant polarization observables, for $p + d$ - and $p + \alpha$ -collisions, in case of inclusive reactions and for some selected exclusive channels.

I. INTRODUCTION

After the static properties as masses, widths and magnetic moment, a good comprehension of the form factors in the ground state as well as in the excited state, is required. In the field of nucleon resonances, in particular, the nature of the nucleonic resonance $P_{11}(1440)$, the *Roper resonance*, with isotopic spin 1/2 and usual spin 1/2 (excited nucleon) is still not well known. This resonance was discovered during the phase shift analysis of data on πN -scattering [1], it is predicted by most of the quark models, but never directly observed.

The measurement of transition nucleon form factors is mostly based on studies with electromagnetic probes where one measures the different polarization observables for meson electroproduction processes. However, due to the isospin nonconservation in the electromagnetic interaction, a photon does not have a definite value of isospin. Therefore it is not possible to separate the isoscalar and isovector contributions, using only data about inclusive $\gamma^* p$ - and $\gamma^* n$ -collisions, but, due to the isoscalar selectivity of the deuteron probe (in the framework of a t -channel ω -exchange approximation for the process $\vec{d} + p \rightarrow d + X$), it is possible to determine the isoscalar amplitude separately. In this respect hadronic processes may be considered as the necessary and complementary tools to study the isotopic structure of $\gamma^* N$ -interactions.

The study of N^* -structure by hadronic probe has known advantages with respect to real or virtual photons: large values of cross sections, advanced technics of high intensity \vec{d} and \vec{p} beams, polarized targets and polarimeters, absence of problem of radiative corrections, natural selection of isoscalar N^* -excitation, if one chooses isoscalar particles, as d or α . On the other hand, the reaction mechanism is not well known *a priori*. The ω -exchange seems to be the best mechanism to describe the N^* -excitation in dp - or $d\alpha$ -collisions: the ωNN coupling constant is large, a spin one exchange allows to obtain very specific polarization phenomena and energy-independent cross section. In the framework of this mechanism, it is possible to predict all observables for the reaction $d + p \rightarrow d + X$ in terms of deuteron electromagnetic form factors and isoscalar form factors of $N \rightarrow N^*$ transitions [2,3].

In this contribution we will recall the properties of the $d + p \rightarrow d + X$ and $\alpha + p \rightarrow \alpha + X$ reactions in the framework of ω - (σ -, η -) exchange and derive the relevant polarization observables.

II. THE REACTION $d + p \rightarrow d + X$

The differential cross section has been measured in the past at incident energy $E_d=1.6$ GeV [4] and more recently at $E_d=2.3$ GeV, at the Saturne accelerator [5]. Polarization observables have been measured at Saturne [5] and Dubna [6]. The existing data on the differential cross section for this reaction show the presence of at least two mechanisms in the intermediate energy region ($2 \leq E_{kin} \leq 10$ GeV). One is the coherent excitation of d with pion production (Fig. 1a), which results in a Deck peak [7], in the energy spectrum of the scattered deuterons. The isotopic spin of this peak is $I = 1/2$, but the spin \mathcal{J} and the space parity P of the Deck peak may not have a unique value. The Deck effect decreases when the energy of the colliding particles increases, while the role of a second mechanism, the direct

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N^* – excitation, must become more important. This can be described by a t –channel exchange by mesonic states (Fig. 2b), with $I = 0$ and with different \mathcal{J}^P : σ , η , ω ...

The tensor analyzing powers, which are strongly negative, show a large similarity with the tensor polarization data measured in ed elastic scattering [8], therefore suggesting the ω –meson being the most probable mediator.

An extended version of a model based on t –channel ω –exchange has been published in [2,3]. Let us recall here two main results: the linearity of the cross section as a function of a kinematical variable y , defined below, and a factorization of the tensor analyzing power in a function of the deuteron and the N^* electromagnetic form factors.

A. General structure of the differential cross-section

The differential cross section, corresponding to diagram 1b (with ω –exchange), can be written in the following form:

$$\frac{d^2\sigma}{dt dw^2} = N[y^2 + a(t, w^2)], \quad (2.1)$$

where: $y = \frac{p_1 \cdot p_2 - k \cdot p_2/2}{mM}$, $t = (p_1 - p_3)^2$ and w is the invariant effective mass of X , in the process $d + p \rightarrow d + X$. Here m (M) is the proton (deuteron) mass. The notation of four-momenta is illustrated in Fig. 1b.

The linear y^2 –dependence of the differential cross section is a direct consequence of the spin one ω –exchange in the t –channel. In a general case the inclusive scattering process $p(d, d)X$ is characterized by 3 independent variables: t, w^2 and s , $s = (p_1 + p_2)^2$. The dependence of the cross section from t and w^2 has a dynamical character and is described by the corresponding SF's for the $dd\omega$ – and ωpX – vertexes.

The linearity of the dependence (1) of the double differential cross section on y^2 can be experimentally tested by a measurement of the differential cross section, at three different energies of the initial deuteron (3 different values of s), at fixed values of t and w^2 . This linearity is a direct consequence of the ω –exchange mechanism, and such measurement would be an experimental test of the validity of the ω –exchange mechanism, equivalent to the Rosenbluth fit for electron-hadron scattering. One can mention that the Rosenbluth fit allows also to separate the contributions of longitudinal and transversal photon polarizations to the differential cross section of any process $e^- + A \rightarrow e^- + X$. Similarly, the study of the y^2 –linearity of the double differential cross section for processes $d + p \rightarrow d + X$ will allow to separate two different structure functions. At the limit $s \gg w^2$, $|t|$, we have:

$$\frac{d^2\sigma}{dt dw^2} = \frac{a(t, w^2)}{32 \pi^2}, \quad s \rightarrow \infty, \quad (2.2)$$

i.e. the differential cross section becomes s –independent, as it is expected for a t –channel exchange of a spin-one meson. In case of η – or σ –exchange the cross section has to decrease with s according to:

$$\frac{d^2\sigma}{dt dw^2} = \frac{f(t, w^2)}{s^2}. \quad (2.3)$$

These different properties should help to sign experimentally the ω –exchange contribution.

B. The tensor analyzing power

The predictions for this observable, in case of ω –, η – or σ –exchange are very different and are given below.

σ –exchange

The $dd\sigma$ –vertex can be described, in the general case, in terms of two independent form factors with the following spin structures: $g_0(t)\vec{U}_1 \cdot \vec{U}_2^*$ and $g_2(t)\hat{k} \cdot \vec{U}_1 \hat{k} \cdot \vec{U}_2^*$, where \hat{k} is the unit vector along the three-momentum transfer, $\vec{U}_1(\vec{U}_2)$ is the vector of polarization of the initial (final) deuteron and $g_0(t)$ and $g_2(t)$ are form factors related to the deuteron form factors.

The tensor analyzing power is:

$$T_{20} = -\sqrt{2}g_2(t) \frac{2g_0(t) + g_2(t)}{3g_0^2(t) + g_2^2(t) + 2g_0(t)g_2(t)}. \quad (2.4)$$

It is possible to induce tensor analyzing power even in the case of σ –exchange (spin 0 particle), only if one takes into account high order effects (interaction with derivatives) which are characterized by $g_2(t)$.

In this case the $dd\eta$ -vertex is characterized by a single spin structure, $g_1(t)\hat{k}\cdot\vec{U}_1\times\vec{U}_2^*$, and using the $NN^*\eta$ -vertex in the form: $\chi_2^\dagger\vec{\sigma}\cdot\hat{k}\chi_1$, one can find $T_{20} = \frac{1}{\sqrt{2}}$, positive and t -independent. The form factor $g_1(t)$ of the $dd\eta$ -vertex is proportional to the magnetic form factor of the deuteron (in the impulse approximation).

The tensor analyzing power in $d + p \rightarrow d + X$, T_{20} , can be written in terms of the deuteron electromagnetic form factors as:

$$T_{20} = -\sqrt{2} \frac{V_1^2 + (2V_0V_2 + V_2^2)r(t)}{4V_1^2 + (3V_0^2 + V_2^2 + 2V_0V_2)r(t)}, \quad (2.5)$$

where $V_0(t)$, $V_1(t)$ and $V_2(t)$ are linear combinations of the standard electric, G_e , magnetic G_m and quadrupole G_q deuteron form factors. The ratio r characterizes the relative role of longitudinal and transversal isoscalar excitations in the transition $\omega + N \rightarrow X$. It can be written, for the excitation of any nucleon resonance N^* , as follows:

$$r(t) = \frac{|A_S^p + A_S^n|^2}{|A_{1/2}^p + A_{1/2}^n|^2 + |A_{3/2}^p + A_{3/2}^n|^2} \equiv \sigma_L(t)/\sigma_T(t), \quad (2.6)$$

where $A_S^{p,n}$ is the longitudinal form factor, $A_{1/2}^{p,n}$ and $A_{3/2}^{p,n}$ are the two possible transversal form factors, corresponding to total $\gamma^* + N$ -helicity equal to $1/2$ and $3/2$, for proton and neutron.

The excitation of overlapping resonances with finite values of widths, introduces a w -dependence in r as follows:

$$r(t) \rightarrow r(t, w) = \frac{\sum_i \sigma_{L,i}(t) B_i(w) C_i}{\sum_i \sigma_{T,i}(t) B_i(w) C_i}, \quad (2.7)$$

where $B_i(w)$ is a Breit-Wigner function for the i -th N^* -resonance with a definite normalization:

$$C_i^{-1} = \int_{m+m_\pi}^{\infty} dw B_i(w), \quad (2.8)$$

where m_π is the pion mass.

The interference due to the excitation of different resonances does not appear in the inclusive processes. It is contained in the angular distribution of the decay products (for example π -production), on which an integration is implicitly performed in the previous formulas.

From Eq. (5) one can see that all information about the ωNN^* -vertex is contained in the function r only. A zero value of r results in a t - and w -independent value for T_{20} , namely $T_{20} = -1/\sqrt{8}$, for any value of the deuteron electromagnetic form factors. The ratio r is calculated using the collective string model in [9–11], assuming $SU_{sf}(6)$ symmetry, including the contributions of the following resonances: $N_{11}(1440)$, $S_{11}(1535)$, $D_{13}(1520)$ and $S_{11}(1650)$, which are overlapping in this energy region.

Of these four resonances only the Roper resonance has a nonzero isoscalar longitudinal electromagnetic form factor. The isoscalar longitudinal amplitudes of $S_{11}(1535)$ and $D_{13}(1520)$ excitations vanish because of spin-flavor symmetry, while both isoscalar and isovector longitudinal couplings of $S_{11}(1650)$, $D_{15}(1675)$ and $D_{13}(1700)$ excitations vanish identically.

In Fig. 2 we report the theoretical predictions for T_{20} , using Eqs. (5-6), together with the existing experimental data. In such approximation T_{20} is a universal function of t only, without any dependence on the initial deuteron momentum. The experimental values of T_{20} for $p(\vec{d}, d)X$ [5,6], for different momenta of the incident beam are shown as open symbols. These data show a scaling as a function of t , with a small dependence on the incident momentum, in the interval 3.7-9 GeV/c. On the same plot the data for the elastic scattering process $e^- + d \rightarrow e^- + d$ [8] are shown (filled stars).

These different data show a very similar behavior: negative values, with a minimum in the region $|t| \simeq 0.35 \text{ GeV}^2$ and their value increase toward zero at larger $|t|$. The lines are the result of the ω -exchange model for the $d+p \rightarrow d+X$ process: for $r = 0$ (dashed-dotted line), the calculation based on [9] for the Roper excitation only is represented by the dotted line and for the excitation of all the four resonances by the full line. The deuteron electromagnetic form factors have been taken from [12], a calculation based on relativistic impulse approximation, and they reproduce well the T_{20} -data for ed elastic scattering [8]. When $r \gg 0$ or if the contribution of the deuteron magnetic form factor $V_1(t)$ is neglected, then T_{20} does not depend on the ratio r , and coincides with t_{20} for the elastic ed -scattering (with the same approximation).

From Fig. 2 it appears that the t -behavior of T_{20} is very sensitive to the value of r especially at relatively small r , $r \leq 0.5$. The values of r , predicted by model [9], give a very good description of the data, when taking into account the contribution of all four resonances. These data, in any case, exclude a very small value of r , $r \ll 0.1$ as well as very large values of r . Such sensitivity of T_{20} for $d + p \rightarrow d + X$ to the ratio of the corresponding isoscalar form factors of the N^* -excitation gives an evident indication of the excitation of the Roper resonance in this process. The predicted dependence of the ratio $r(t, w)$ and of T_{20} , on the initial deuteron momentum is not so large, in agreement with experimental data.

This model for $d + p \rightarrow d + X$ may be improved, taking into account for example, other meson exchanges, or the effects of the strong interaction in initial and final states. However the corrections that can be added to the model presented here are strongly model and parameter dependent, and are not justified by the existing experimental data.

Let us note in this connection, that, in the considered model, all T-even polarization observables are nonzero and large in absolute value. This is an intrinsic property of ω -exchange. But all T-odd polarization effects cancel, because we neglected the strong interaction in initial and final states. In case of collinear kinematics, all one spin T-odd polarization observables vanish, in any model. The most simple T-odd polarization observable, which exists in the general case for the collinear kinematics, corresponds to the following correlation coefficient: $\hat{k} \cdot \vec{P} \times \vec{Q}$, $Q_a = Q_{ab}k_b$, where \vec{P} is the proton polarization and Q_{ab} is the deuteron tensor polarization. A measurement of these observables will give a direct information on the presence and intensity of the final or initial strong interaction. The tensor analyzing power T_{20} , being a T-even observable, is less sensitive to such effects.

C. Coefficients of polarization transfer in $\vec{d} + p \rightarrow \vec{d} + X$

Let us calculate the (vector) transfer polarization coefficients $K_a^{a'}$ (with $a, a' = x, y$ or z) from initial to final deuterons, in the processes $\vec{d} + p \rightarrow \vec{d} + X$, in the framework of ω -exchange. Using the corresponding parametrization of the ωdd -vertex, one can find the following formulas, for the nonzero polarization transfer coefficients:

$$K_y^{y'} = K_x^{x'} = \frac{3}{2} \frac{V_1^2 + (V_0 V_2 + V_0^2)r(t)}{4V_1^2 + (3V_0^2 + V_2^2 + 2V_0 V_2)r(t)}, \quad (2.9)$$

$$K_z^{z'} = \frac{3}{2} \frac{V_1^2 + V_0^2 r(t)}{4V_1^2 + (3V_0^2 + V_2^2 + 2V_0 V_2)r(t)}. \quad (2.10)$$

For $r = 0$ we obtain simply: $K_x^{x'} = K_y^{y'} = K_z^{z'} = \frac{3}{8} = 0.375$: all coefficients are positive and t -independent. As in the case of T_{20} the ratio $r(t)$ contains all the information about the properties of $\omega p \rightarrow X$ vertexes. From Fig. (3), one can see that $K_y^{y'}$ has a strong q and r dependence. The point where all lines for different r are crossing is due (as for T_{20}) to a special combination of V_0 and V_2 (see the numerator of Eq. (6)). At $q = 0$, $K_y^{y'} = 1/2$, for any value of r , as $V_1 = V_2 = 0$. The largest sensitivity to r is in the region $q \geq 3 \text{ fm}^{-1}$ and the position of zero crossing is strongly r -dependent.

III. POLARIZATION PHENOMENA IN $\vec{p} + d \rightarrow \vec{p} + M^0 + d$, $M^0 = \sigma$ OR π

The polarization properties of the produced protons in the processes $\vec{p} + d \rightarrow \vec{p} + M^0 + d$, $M^0 = \sigma$, η or π^0 depend essentially on the kind of the produced meson and on the quantum numbers (\mathcal{J}^P) of the nucleonic resonance in the intermediate state: $p + d \rightarrow N^*(\mathcal{J}^P) + d \rightarrow p + M^0 + d$. Let us consider the case of Roper excitation, with $\mathcal{J}^P = \frac{1}{2}^+$.

A. ω -exchange

The nonzero polarization transfer coefficients for σ -production: $\vec{p} + d \rightarrow \vec{p} + \sigma + d$ are :

$$K_y^{y'} = K_x^{x'} = \frac{\mathcal{R}}{4 + \mathcal{R}}, \quad K_z^{z'} = -\frac{-4 + \mathcal{R}}{4 + \mathcal{R}}, \quad \mathcal{R}^{-1} = \frac{V_1^2}{(3V_0^2 + V_2^2 + 2V_0 V_2)r(t)}, \quad (3.1)$$

i.e. these coefficients depend only on the momentum transfer t .

On the other hand a dependence from the M^0 -production angle is contained in the decays $N^* \left(\frac{1}{2}^+ \right) \rightarrow p + \pi^0$ (or $p + \eta$), with P -wave production of the pseudoscalar meson. In this case we obtain that all $K_a^{a'}$ coefficients allowed by the P -invariance of strong interaction are nonzero:

$$K_y^{y'} = -\frac{\mathcal{R}}{4 + \mathcal{R}}, \quad (3.2)$$

$$K_x^{x'} = (1 - 2\cos^2\theta_\pi) \frac{\mathcal{R}}{4 + \mathcal{R}}, \quad (3.3)$$

$$K_z^{z'} = (1 - 2\cos^2\theta_\pi) \frac{4 - \mathcal{R}}{4 + \mathcal{R}}, \quad (3.4)$$

$$K_x^{z'} = (\sin 2\theta_\pi) \frac{-4 + \mathcal{R}}{4 + \mathcal{R}}, \quad (3.5)$$

$$K_z^{x'} = (\sin 2\theta_\pi) \frac{\mathcal{R}}{4 + \mathcal{R}}, \quad (3.6)$$

where we used a coordinate system with the z -axis along the momentum transfer \vec{k} , $y \parallel \vec{n}$, with $\hat{n} = \vec{k} \times \vec{q} / |\vec{k} \times \vec{q}|$, (\vec{q} is the meson 3-momentum) and $x \parallel \vec{n} \times \vec{k}$.

An important property of the ω -exchange model is the universal dependence of all the $K_a^{a'}$ from t , through the ratio $\mathcal{R}(t)$. An experimental check of the θ_π -dependence of the polarization transfer coefficient would be a signature of the validity of this model.

B. σ -exchange

In the case of σ -exchange it is possible to analyze the spin transfer coefficients in a general form, with a model independent parametrization, for two possible generalized vertexes: $\sigma + N \rightarrow \sigma + N$ and $\sigma + N \rightarrow \pi^0 + N$. The spin structure of the corresponding matrix elements can be written as follows:

$$\begin{aligned} F(\sigma N \rightarrow \sigma N) &= \chi_2^\dagger (A + i\vec{\sigma} \cdot \hat{n} B) \chi_1, \\ F(\sigma N \rightarrow \pi N) &= \chi_2^\dagger (C\vec{\sigma} \cdot \hat{k} + D\vec{\sigma} \cdot \hat{q}) \chi_1, \end{aligned}$$

where A , B , C , and D are the corresponding amplitudes, depending from three independent kinematical variables: w (the invariant effective mass of the produced MN -system), $\cos\theta_\pi = \hat{k} \cdot \hat{q}$ and t . In case of σ -exchange the spin transfer coefficients depend only from the ratio $r_g(t) = g_2(t)/g_0(t)$ of the form factors of the $dd\sigma$ -vertex. For the nonzero (diagonal) coefficients the following expressions hold:

$$K_y^{y'} = K_x^{x'} = \frac{3(1 + r_g)}{2(3 + 2r_g + r_g^2)}, \quad K_z^{z'} = \frac{3}{2(3 + 2r_g + r_g^2)}, \quad (3.7)$$

where the z -axis is along the 3-momentum transfer \vec{k} . In this case the observables $K_a^{a'}$ contain the same information as T_{20} , Eq. (5). Moreover these observables are connected by the following relation:

$$T_{20} - \frac{3}{\sqrt{2}} K_z^{z'} + \frac{9}{2\sqrt{2}} (K_y^{y'})^2 = 0. \quad (3.8)$$

The polarization properties of both protons, in the initial and final states, do not depend from the structure of the $dd\sigma$ -vertex, but depend on the amplitudes A and B for σ -production (or on the amplitudes C and D for π production).

The vector polarization \vec{P} of the protons, produced in the collision of unpolarized particles has to be different from zero, in the general case:

$$\vec{P} = \hat{n} \frac{2\text{Im } AB^*}{|A|^2 + \sin^2\theta_\pi |B|^2}$$

for σ -production, and

$$\vec{P} = \hat{n} \frac{2\text{Im } CD^*}{|C|^2 + 2\cos\theta_\pi \text{Re } CD^* + |D|^2}$$

in case of $\pi(\eta)$ -production. The analyzing powers in $\vec{p} + d \rightarrow p + M + d$ are determined by the evident relation: $\mathcal{A}_y = P$.

The dependence of the polarization \vec{P}_2 of the produced proton on the polarization \vec{P}_1 of the proton beam is described by the following general formulas:

$$\vec{P}_2 = \vec{P}_1 \frac{|A|^2 - \sin^2\theta_\pi |B|^2}{|A|^2 + \sin^2\theta_\pi |B|^2} + 2\hat{n}(\hat{n} \cdot \vec{P}_1) \frac{|B|^2}{|A|^2 + \sin^2\theta_\pi |B|^2} - 2(\hat{n} \times \vec{P}_1) \frac{\text{Re} AB^*}{|A|^2 + \sin^2\theta_\pi |B|^2},$$

for σ -production, and

$$\vec{P}_2 = -\vec{P}_1 + \frac{2\hat{k}(\hat{k} \cdot \vec{P}_1)|C|^2}{|C|^2 + 2\text{Re} CD^* \cos\theta_\pi + |D|^2} + 2\hat{k}(\hat{k} \cdot \vec{P}_1) \frac{|D|^2}{|C|^2 + 2\text{Re} CD^* \cos\theta_\pi + |D|^2}$$

for $\pi^0(\eta)$ -production.

From these formulas one derives the following expressions for the coefficients $K_a^{a'}$:

1. σ -exchange with σ -production ($\vec{p} + d \rightarrow \vec{p} + \sigma + d$):

$$\begin{aligned} K_y^{y'} &= +1, \\ K_x^{x'} &= K_z^{z'} = \frac{|A|^2 - \sin^2\theta_\pi |B|^2}{|A|^2 + \sin^2\theta_\pi |B|^2}, \\ K_x^{z'} &= K_z^{x'} = 2\sin^2\theta_\pi \frac{\text{Re} AB^*}{|A|^2 + \sin^2\theta_\pi |B|^2}. \end{aligned}$$

2. σ -exchange with π^0 -production ($p + d \rightarrow p + \pi^0(\eta) + d$):

$$\begin{aligned} K_y^{y'} &= -1, \\ K_x^{x'} &= -1 + \frac{2\sin^2\theta_\pi |D|^2}{|C|^2 + 2\text{Re} CD^* \cos\theta_\pi + |D|^2}, \\ K_x^{z'} &= K_z^{x'} = \frac{\sin^2\theta_\pi |D|^2}{|C|^2 + 2\text{Re} CD^* \cos\theta_\pi + |D|^2}. \end{aligned}$$

C. Case of interfering resonances

The formulas, previously derived, are general and they can be applied to any resonance. The amplitudes A , B , C and D depend on the quantum numbers of the \mathcal{J}^P of the excited nucleonic resonances. Let us consider two cases:

(a) RR excitation, with $\mathcal{J}^P = \frac{1}{2}^+$,

(b) $N^*(1535)$ excitation, with $\mathcal{J}^P = \frac{1}{2}^-$.

1. σ -exchange with σ -production ($p + d \rightarrow p + \sigma + d$):

We obtain the following amplitudes:

(a) : $A = f^{(+)}(t)\mathcal{H}^{(+)}(w)g_\sigma^{(+)}$, $B = 0$.

(b) : $A = \cos\theta_\pi B$, $B = f^{(-)}(t)\mathcal{H}^{(-)}(w)g_\sigma^{(-)}$,

where $g_\sigma^{(\pm)}$ is the constant for the decay $N^* \left(\mathcal{J}^P = \frac{1}{2}^\pm \right) \rightarrow N + \sigma$, $\mathcal{H}^{(\pm)}(w)$ is the corresponding propagator to describe the Breit Wigner behavior:

$$\mathcal{H}^{(\pm)}(w) = \frac{1}{w - m^* - i\frac{\Gamma}{2}}$$

for the excitation of N^* with mass m^* and total width Γ ; $f^{(\pm)}$ are form factors of the vertexes $p \rightarrow N^* + \sigma$.

2. σ -exchange with π^0 -production ($p + d \rightarrow p + \pi^0(\eta) + d$):

$$(a) : C = 0, D = f^{(+)}(t)\mathcal{H}^{(+)}(w)g_{\pi}^{(+)}.$$

$$(b) : C = f^{(-)}(t)\mathcal{H}^{(-)}(w)g_{\pi}^{(-)}, D = 0,$$

where $g_{\pi}^{(\pm)}$ is the constant for the decay $N^* \left(\mathcal{J}^P = \frac{1}{2}^{\pm} \right) \rightarrow N + \pi$.

We can conclude that any single resonance induces zero analyzing power in the processes $\vec{p} + d \rightarrow p + \sigma(\pi) + d$. Nonzero analyzing powers result from the interference of different resonances contributions. On the other hand the vector polarization transfer coefficients are nonzero for a single resonance and depend strongly on \mathcal{J}^P and on the produced meson. Using the general formulas obtained above, we find for the polarizations:

1. σ -exchange with σ -production ($\vec{p} + d \rightarrow \vec{p} + \sigma + d$):

$$(a) : \vec{P}_2 = \vec{P}_1.$$

$$(b) : \vec{P}_2 = (-1 + 2\cos^2\theta_{\pi})\vec{P}_1 + 2\hat{n}(\hat{n} \cdot \vec{P}_1) + 2\cos\theta_{\pi}\vec{P}_1 \times \hat{n}.$$

2. σ -exchange with π^0 -production ($\vec{p} + d \rightarrow \vec{p} + \pi^0(\eta) + d$):

$$(a) : \vec{P}_2 = -\vec{P}_1 + 2\hat{q}(\hat{q} \cdot \vec{P}_1).$$

$$(b) : \vec{P}_2 = -\vec{P}_1 + 2\hat{k}(\hat{k} \cdot \vec{P}_1).$$

D. Polarization effects in case of interfering resonances

In conclusion of this section we give the results for the polarization properties of protons in processes $p + d \rightarrow p + M^0 + d$, induced by the **interference of two nucleonic resonances** with $\mathcal{J}^P = \frac{1}{2}^{+}$, and $\mathcal{J}^P = \frac{1}{2}^{-}$, in the framework of the σ -exchange model.

1. σ -production ($\vec{p} + d \rightarrow \vec{p} + \sigma + d$):

$$(a) : \vec{P}_2 = \hat{n}\mathcal{A}_{\sigma}, \quad \frac{d\sigma}{d\omega} = \left(\frac{d\sigma}{d\omega} \right)_0 (1 + \mathcal{A}_{\sigma}\hat{n} \cdot \vec{P}_1), \quad \mathcal{A}_{\sigma} \simeq 2\text{Im } cd^*,$$

$$(b) : \vec{P}_2 \simeq 2\hat{n} \times \vec{P}_1 \text{Re } cd^*, \text{ where:}$$

$$c = f^{(-)}(t)\mathcal{H}^{(-)}(w)g_{\sigma}^{(-)},$$

$$d = f^{(+)}(t)\mathcal{H}^{(+)}(w)g_{\sigma}^{(+)} + \cos\theta_{\pi}c.$$

2. $\pi^0(\eta)$ -production ($\vec{p} + d \rightarrow \vec{p} + \pi^0(\eta) + d$):

$$(a) : \vec{P}_2 = \hat{n}\mathcal{A}_{\pi}, \quad \frac{d\sigma}{d\omega} = \left(\frac{d\sigma}{d\omega} \right)_0 (1 + \mathcal{A}_{\pi}\hat{n} \cdot \vec{P}_1), \quad \mathcal{A}_{\pi} \simeq 2\text{Im } ab^*,$$

$$(b) : \vec{P}_2 \simeq 2 \left[-\cos\theta_{\pi}\vec{P}_1 + \hat{q}\hat{k} \cdot \vec{P}_1 + \hat{k}\hat{q} \cdot \vec{P}_1 \right] \text{Re } ab^*,$$

where we used the following notations:

$$a = f^{(+)}(t)\mathcal{H}^{(+)}(w)g_{\pi}^{(+)},$$

$$b = f^{(-)}(t)\mathcal{H}^{(-)}(w)g_{\pi}^{(-)}.$$

These formulas show that the T-odd analyzing power in $\vec{p} + d$ -collisions (or the polarization of the final proton, produced in the collision of unpolarized particles) are especially sensitive to the interference of different nucleonic resonances.

IV. REMARKS ON POLARIZATION OBSERVABLES IN THE PROCESS $p + \alpha \rightarrow p + \pi^0 + \alpha$

The reaction $p + \alpha \rightarrow p + \pi^0 + \alpha$ has been considered a good tool for the study of the Roper resonance [13] as the α particle, is known to be a good isoscalar probe. Calculations on this reaction have been done in [14] and polarization observables analyzed in [15], in framework of σ -exchange.

Two main features characterize this reaction: it is a three body reaction, in the final state, with non coplanar kinematics and only protons have nonzero spin. The acoplanarity is related to the following combination of 3-momenta

$$: a = \frac{\vec{q} \cdot \vec{p}_1 \times \vec{p}_2}{E_1 E_2 E_{\pi}}, \text{ where } \vec{p}_1 \text{ and } \vec{p}_2 \text{ are the three momenta of the initial and final proton, } \vec{q} \text{ is the 3-momentum of}$$

the produced pion and E_1 , E_2 , E_π the corresponding energies. This combination is present in the full set of the 5 independent kinematical variables which are necessary for the complete description of a process $1 + 2 \rightarrow 3 + 4 + 5$.

The variable a is connected with the azimuthal angle ϕ - between two different reaction planes: the scattering plane of the proton (i.e. the plane defined by the 3-momenta \vec{p}_1 and \vec{p}_2) and the plane defined by the pion three-momentum \vec{q} and the transferred momentum $\vec{p} = \vec{p}_1 - \vec{p}_2$. The polarization of the final proton can be parametrized in the following general form:

$$\vec{P} = \hat{n}P_n + a(\hat{m}P_m + \hat{k}P_k), \quad (4.1)$$

where the unit vectors \hat{m} , \hat{n} and \hat{k} are redefined in this section as: $\hat{n} = \vec{p}_1 \times \vec{p}_2 / |\vec{p}_1 \times \vec{p}_2|$, $\hat{k} = \vec{p}_1 / |\vec{p}_1|$, $\hat{m} = \hat{n} \times \hat{k}$. P_n , P_m and P_k are the three independent and nonzero components of the final proton polarization vectors. If the P-invariance of the strong interaction holds, the matrix element is described by the following general parametrization (in the CMS of the considered reaction):

$$\mathcal{M} = \chi_2^\dagger \left[\vec{\sigma} \cdot \vec{m} f_1 + \vec{\sigma} \cdot \vec{k} f_2 + a \left(i \tilde{f}_1 + \vec{\sigma} \cdot \vec{n} \tilde{f}_2 \right) \right] \chi_1, \quad (4.2)$$

where χ_1 and χ_2 are the 2-component spinors of the protons in the initial and final states; f_1 , f_2 , \tilde{f}_1 and \tilde{f}_2 are the scalar independent amplitudes for $p + \alpha \rightarrow p + \pi^0 + \alpha$, which are functions of the 5 kinematical variables, defined above.

Nine coefficients of polarization transfer, D_{ij} can be defined in terms of the scalar amplitudes f_i and \tilde{f}_i . Let's focus on D_{nn} .

$$D_{nn} \left(\frac{d\sigma}{d\omega} \right)_0 = -|f_1|^2 - |f_2|^2 + a^2 \left(|\tilde{f}_1|^2 + |\tilde{f}_2|^2 \right).$$

In collinear kinematics only a single and specific spin structure is present: all polarization phenomena can be predicted exactly, in a model independent form:

$$D_{mm} = D_{nn} = -D_{kk} = -1,$$

and all other polarization observables have to be identically zero, due to the azimuthal symmetry of the collinear kinematics.

In the general case, two main mechanism contribute to the matrix element for $p + \alpha \rightarrow p + \pi^0 + \alpha$, one related to the projectile and the other to the target excitation. Following [15], the Deck mechanism results from π -exchange, but the Roper excitation is induced by σ -exchange. In this hypothesis, one can show that both non-coplanar amplitudes \tilde{f}_1 and \tilde{f}_2 are zero for both mechanisms, taking into account the most general properties of $\pi\alpha$ -scattering (for the Deck mechanism) and of the process $\sigma + N \rightarrow N + \pi$ (for the Roper excitation). From the spin structure given above, one can deduce that $D_{nn} = 1$, in the whole region of kinematical variables (for coplanar and non-coplanar kinematics). The polarization of the final proton has only one non-zero component, in the \hat{n} -direction, i.e. along the normal to the proton scattering plane. and its sign and absolute value depend on the relative role of the considered mechanisms i.e. to the details of the corresponding amplitudes.

This 'coplanar-like' behavior of σ - and π - exchanges in $p + \alpha \rightarrow p + \pi^0 + \alpha$ is related to the fact that these mediators are spinless particles. Such exchanges can not connect different reaction planes. This conclusion does not depend on details, approximations, values of the constants or shape of form factors which are typically taken in the numerical applications, because it is based only on the value of the spin of the mediators.

Earlier we suggested that the ω - exchange is the most probable mechanism for the Roper excitation, in this energy range [2]. The most important difference with respect to σ -exchange is due to the spin and has evident implications for the polarization phenomena: a vector particle exchange induces all four amplitudes different from zero, in the general case.

Future experimental data on polarization observables for $p + \alpha \rightarrow p + \pi^0 + \alpha$ will constitute a crucial test in order to disentangle the mechanisms involved.

V. CONCLUSION

Let us summarize the main results of the previous analysis of polarization phenomena for the Roper excitation in dp - and αp -collisions, in the framework of ω - exchange, that we consider as the most probable mechanism for these processes.

- Polarization effects are rich in information, especially for correlation experiments, like $d + p \rightarrow d + p + M$, where M is π^- , η - or σ -meson.
- The t -dependence of the existing data on tensor analyzing power in $d + p \rightarrow d + X$ is sensitive to the value of the spin of exchanged particles and agrees very well with the model based on ω -exchange.
- The contributions of the four overlapping resonances, $N_{11}(1440)$, $S_{11}(1535)$, $D_{13}(1520)$ and $S_{11}(1650)$, have to be taken into account, to reproduce the t -dependence of the tensor analysing power in $d + p \rightarrow d + X$. The longitudinal form factor of the Roper excitation is essential for a correct description of experimental data.
- Polarization phenomena in $p + d \rightarrow p + d + M$ are sensitive to the type of the produced meson M and to the quantum numbers of mediators.
- The acoplanarity of processes with three particles in the final state, introduces many new interesting aspects in the analysis of the spin structure of matrix elements and polarization phenomena.
- The spin structure of matrix element for $p + \alpha \rightarrow p + \pi + \alpha$ is characterized by four scalar amplitudes -with two non-coplanar amplitudes, so that the polarization vector of scattered protons has, in general, all three nonzero components.

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FIG. 1. Possible mechanisms for $d + p \rightarrow d + \pi + N$: (a) deuteron excitation; (b) proton excitation.

FIG. 2. Experimental data on T_{20} for $d + p \rightarrow d + X$ at incident momenta of 3.75 GeV/c (open diamond) [5] 5.5 GeV/c (open circles), 4.5 GeV/c (open squares), and 9 GeV/c (open triangles) [6]. The prediction of the ω -exchange model for $r = 0$ is represented by the dashed-dotted line. The calculation with r from [9] is represented by the dotted line for the Roper excitation and by the solid line for the excitation of the four resonances mentioned in the text. The t_{20} data from ed elastic scattering (filled stars) are from [8].

FIG. 3. Vector polarization transfer coefficient $K_y^{y'}$ as a function of q , for $d + p \rightarrow d + X$. The calculation with r from [9] is represented by the dotted line for the Roper excitation and by the solid line for the excitation of the four resonances mentioned in the text.





